A Discrete-Time Model of Nonlinear Non-Autonomous Systems

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Abstract—A discretization method is proposed for continuous time, non-autonomous, and nonlinear systems. The concept of continualization is used to derive a sufficient condition for a given discrete-time system to be an exact discretization of a continuous-time system. The proposed discretization method is based on an approximate solution to this condition, which is computed using Peano-Baker series. As an example, an inverted pendulum subjected to high-frequency excitation is considered. Simulation results show that the proposed method has good performances even with a relatively large sampling interval.

I. INTRODUCTION

Numerical simulations and on-line computations of nonlinear continuous-time systems, for which no complete solutions are known, require conversion of their governing equations into ones in discrete-time formats. A number of researchers have worked on this topic in a variety of fields, including engineering systems and control [1-4]. While accurate discretization methods, such as Runge-Kutta schemes, are available for off-line simulations, those that lead to on-line computable algorithms, such as those used for digital control or prediction, are still relatively rare. Some discretization methods, such as multi-step schemes, make the order of difference equations higher than that of the original differential equations, and can lead to numerical instabilities unless a sufficiently small sampling interval is used. The simplest and widely-applicable discretization method for on-line computation is the forward-difference model. However, this model has a performance that is usually poor unless a sufficiently small sampling interval is used. In some cases this model yields dynamics that differ from those of continuous-time systems even for a small sampling interval. While some better discretization methods have been proposed [5-6], they are mainly intended for autonomous nonlinear systems. Non-autonomous systems contain time-varying parameters and functions that cannot be treated properly as constants within a sampling interval. This makes the relationship between continuous-time systems and their discrete-time models more complicated, and the discretization methods for non-autonomous systems more difficult, even for linear systems. Studies on this issue have been limited practically to linear [7] and a few nonlinear [8] systems. Therefore, at present, the forward difference method is virtually the only on-line computable discretization method that is applicable to a wide range of nonlinear systems including non-autonomous cases. However, due to its limited performance, more work is desired to develop a more accurate on-line method.

An exact discretization of nonlinear systems governed by a Riccati equation has been developed in [9] using the concept of discrete-time integration gains. Using this concept and introducing another concept of continualization, an approximate method that is applicable to more general autonomous nonlinear systems was proposed in [10]. Its good performances were shown by simulations for Lorenz and van der Pol oscillators. This concept indicated the importance of the relationship between continuous-time systems and discrete-time models and led to a certain sufficient condition. While the integration gain concept is not applicable, the condition equation derived based on the continualization concept is applicable to nonlinear non-autonomous systems. By solving this equation approximately, a discretization method that could be applied to a special class of non-autonomous systems such as forced nonlinear oscillator, where the relation between system states and time varying terms is linear, was developed in [11]. The present paper extends this method to a wider class of non-autonomous systems.

The paper is organized as follows: Section 2 lists some definitions to clarify the meanings of discretization and continualization of signals and systems. Section 3 develops an expression of a continuous-time system based on information of a given discrete-time system, to which the discrete-time model is exact. A discrete-time model is then proposed by solving this expression approximately. Section 4 presents an example of an inverted pendulum subjected to high-frequency excitation and some simulation results. Section 5 provides conclusions.

II. PRELIMINARIES ON DISCRETIZATION AND CONTINUALIZATION

Let a continuous-time model of a non-autonomous nonlinear system be given by the following state space equation:

\[ \frac{d \mathbf{x}(t)}{dt} = \mathbf{\Gamma}(\mathbf{x}(t), t), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \]  

(1)

where \( \mathbf{x} \in \mathbb{R}^n \) is a state vector of continuous-time variable \( t \) and \( \mathbf{\Gamma} \) assumed to be expandable into Taylor series. This implies that \( \mathbf{\Gamma} \) satisfies the Lipschitz condition and (1) has a unique solution for a given initial condition.

For the above continuous-time system, a discrete-time system can be associated. In delta form [12] with a uniform discrete-time period \( T \), it is written as

\[ \delta \mathbf{x}_k = \frac{x_{k+1} - x_k}{T} = \mathbf{\Gamma}(\mathbf{x}_k, T), \quad x_{k_0} = \mathbf{x}_0 \]  

(2)
where \( \mathbf{x}_k = \mathbf{x}(kT) \in \mathbb{R}^n \) is a discrete-time state vector and \( \delta \) the delta operator defined as \( \delta = \frac{(q-1)}{T} \) with \( q \) being the shift-left operator such that \( q\mathbf{x}_k = \mathbf{x}_{k+1} \). It is assumed that the initial time is synchronized such that
\[
t_0 = k_0 T. \tag{3}
\]

Given an appropriate initial condition, the discrete-time equation (2) has a unique solution as long as \( \Gamma \) is defined for each of its arguments [10].

**Definition 1** [Exact Discretization][5]: A discrete-time state \( \mathbf{x}_i \) of system (2) is said to be an exact discretization of a continuous-time state \( \mathbf{x}(t) \) of system (1) if the following relationship holds for any \( k \) and \( T \):
\[
\mathbf{x}_i = \mathbf{x}(kT). \tag{4}
\]

In this case, a discrete-time system, whose state is \( \mathbf{x}_i \), is said to be an exact discrete-time model of a continuous-time system, whose state is \( \mathbf{x}(t) \). ☐

The existence of an exact discrete-time model is guaranteed under the standard assumption of existence of a solution to (2) [5]. The state of an exact discrete-time model satisfies (4) for any \( T \). As \( T \) is changed, \( \mathbf{x}_i \) represents a new discrete-time sequence.

**Definition 2** [Discretization] [10]: The discrete-time state \( \mathbf{x}_i \) is said to be a discretization of the continuous-time state \( \mathbf{x}(t) \) if the following relationship holds for any fixed instant \( \tau \):
\[
\lim_{T \to 0 \atop T \leq <(k+1)T} \mathbf{x}_i = \mathbf{x}(\tau). \tag{5}
\]

Such a discrete-time system is said to be a discrete-time model of the original continuous-time system (1). ☐

A process that can be considered as a sort of inverse operation of discretization is continuousization, which is the role of hold devices. The definition proposed below will play a key role in the development of new discretization methods. It does not have to be on-line computable, but is used only to clarify conditions at the limit of \( T \) approaching zero.

**Definition 3** [Signal Continualization] [11]: Given the discrete-time state \( \mathbf{x}_i \) of (2), the following continuous-time signal \( \mathbf{x}'(t) \) is said to be a continualization of \( \mathbf{x}_i \): in each interval \( kT \leq t < (k+1)T \) with \( \mathbf{x}'(kT) = \mathbf{x}_i \),
\[
\mathbf{x}'(t) = \mathbf{x}'(kT) + (t-kT) \Gamma(\mathbf{x}'(kT),t-kT). \tag{6}
\]

It should be noted that this continuous-time signal is a function of \( t \) and \( T \). Since \( \mathbf{x}'(kT) = \mathbf{x}_i \), the discrete-time state of system (2) is an exact discretization of the continualized signal (6).

**Definition 4** [System Continualization] [11]: A continuous-time system given by
\[
\mathbf{x}'(t) - \mathbf{x}'(kT) = \frac{d}{dt} \left( (t-kT) \Gamma(\mathbf{x}'(kT),t-kT) \right) \tag{7}
\]
where \( \mathbf{x}'(t) \) is generated by (6) in each \( kT \leq t < (k+1)T \), is said to be a continualized system of discrete-time system (2) if \( \Gamma \) satisfies the following:

\begin{enumerate}
  \item \( \frac{\partial}{\partial \mathbf{x}_i} \left( \frac{\partial(\Gamma(\mathbf{x},s))}{\partial s} \right) \) are continuous functions of \( \mathbf{x} \).
  \item \( s \frac{\partial \Gamma(\mathbf{x},s)}{\partial \mathbf{x}_i} \neq -1. \tag{12} \)
\end{enumerate}

III. THE PROPOSED DISCRETIZATION TECHNIQUES

**Theorem 1** [Exact Discretization]: A discrete-time system given by (2) is an exact discrete-time model of system (1) if \( \Gamma \) satisfies the following: In each interval \( kT \leq t < (k+1)T \),
\[
\frac{d}{dt} \left( (t-kT) \Gamma(\mathbf{x}'(kT),t-kT) \right) = \Gamma(\mathbf{x}'(kT) + (t-kT) \Gamma(\mathbf{x}'(kT),t-kT), t). \tag{8}
\]

**Proof**: For the state \( \mathbf{x}'(t) \) to be a solution of (1), \( \Gamma(\mathbf{x}'(kT),t-kT) \) must be such that (7) equals (1); that is, using (6), in each interval,
\[
\frac{d}{dt} \left( (t-kT) \Gamma(\mathbf{x}'(kT),t-kT) \right) = \mathbf{x}'(t) \tag{9}
\]
\[
= \Gamma(\mathbf{x}(t), \Gamma(\mathbf{x}'(kT),t-kT), t) \tag{9}
\]
\[
= \Gamma(\mathbf{x}'(kT) + (t-kT) \Gamma(\mathbf{x}'(kT),t-kT), t), \tag{9}
\]

which is (8). ☐

When (8) can be solved for \( \Gamma \), the exact discrete-time model can be found. When it is solved approximately, an approximate discrete-time model may be obtained. One such model is proposed below, which is applicable to a general class of non-autonomous nonlinear and linear systems, provided they have a Jacobian matrix.

**Theorem 2** [The Proposed Model]: A discrete-time system given by (2), where \( \Gamma \) is chosen as
\[
\Gamma(\mathbf{x}(t), T) = \frac{1}{T} \int_{t-kT}^{t} e^{\frac{\lambda}{T}} \Gamma(\mathbf{x}_i, \lambda + kT) d\lambda \tag{10}
\]
with \( \mathbf{D}\Gamma \) being Jacobian matrix of \( \Gamma \), is a discrete-time model of the continuous-time system given by (1).

**Proof**: Equation (6) with \( \Gamma \) given by (10) holds for a fixed time \( t = \tau \) in each sampling interval such that
\[
\mathbf{x}'(\tau) - \mathbf{x}'(kT) = \frac{\tau - kT}{T \int_{t-kT}^{t} e^{\frac{\lambda}{T}} \Gamma(\mathbf{x}_i, \lambda + kT) d\lambda} \tag{11}
\]
and this also holds at the limit of \( T \) approaching zero while \( k \) is chosen such that \( kT \leq \tau < (k+1)T \) holds. Thus, noting that, for a fixed \( \tau \) and a suitable choice of \( k \),
\[
\lim_{T \to 0} kT = \tau \tag{12}
\]
and \( \mathbf{D}\Gamma \) is finite, the use of l'Hospital's Rule on the right-hand-side of (11) yields
\[
\lim_{T \to +\infty} \left[ \frac{1}{\tau - kT} \int_0^{\tau - kT} e^{s \cdot \xi(T,t)} dt \xi(T,t) + kT \right] = 0
\]

where \( 0 \leq \zeta < T \). Noting that \( \zeta \Gamma(x(T),\zeta) = 0 \) at \( \zeta = 0 \), a solution to the above linear differential equation in \( \zeta \Gamma(x(T),\zeta) \) gives the following continuous-time function [13]:

\[
\zeta \Gamma(x(T),\zeta) = \int_0^T e^{t \cdot \xi(T,t)} \Gamma(x(T),\zeta) d\lambda.
\]

Adopting this form of function, the discrete-time function is obtained as

\[
\Gamma(x(T),\zeta) = \frac{1}{T} \int_0^T e^{t \cdot \xi(T,t)} \Gamma(x(T),\zeta) d\lambda,
\]

which is (10).

**Remark 1**: The proposed discrete-time function (10) can be written as

\[
\Gamma(x(T),\zeta) = \frac{1}{T} \int_0^T e^{t \cdot \xi(T,t)} \Gamma(x(T),\zeta) d\lambda
\]

In view of Definition 2, system (2) with (10) is a discrete-time model of the continuous-time system (1). □

Equation (10) can be derived as follows: When \( \Gamma \) in (8) is expanded into the Taylor series and truncated with the first two terms as

\[
\Gamma(\xi(T,t) + (k+1)T) = \Gamma(\xi(T,t)) + [D \Gamma(\xi(T,t))](\xi(T,t) + (k+1)T)
\]

\[
(8)
\]

eq \Gamma(\xi(T,t)) + \frac{\tau}{(k+1)T} \Gamma(\xi(T,t))
\]

eq. (8) can always be solved and an approximate discrete-time model obtained. That is, for arbitrary \( \xi(T,t) \), (8) can be expressed as

\[
\frac{d}{dt} \left( \frac{\tau}{(k+1)T} \Gamma(\xi(T,t)) \right)
\]

\[
= \frac{\tau}{(k+1)T} \Gamma(\xi(T,t)) + \frac{\tau}{(k+1)T} \Gamma(\xi(T,t))
\]

\[
\frac{d}{dt} \left( \xi(T,t) + (k+1)T \right)
\]

\[
(9)
\]

Defining \( \zeta = \tau - (k+1)T \), (19) can be written as

\[
\frac{d}{d\zeta} \left( \frac{\tau}{(k+1)T} \Gamma(\xi(T,t),\zeta) \right)
\]

\[
= \frac{\tau}{(k+1)T} \Gamma(\xi(T,t),\zeta) + \frac{\tau}{(k+1)T} \Gamma(\xi(T,t),\zeta)
\]

\[
(20)
\]

Remark 3: When (8) can be solved exactly, an exact discrete-time model can be found. For instance for a linear system, the proposed method gives the exact discrete-time model; i.e., for \( \Gamma \) in (1) given as

\[
\Gamma(x(T),\zeta) = \int_0^T e^{t \cdot \xi(T,t)} \Gamma(x(T),\zeta) d\lambda.
\]
\[
\Gamma(\bar{x}(t), t) = \bar{A}(t) \bar{x} + \bar{B}(t) \bar{u}(t),
\]
where \( \bar{A} \) is a system matrix of compatible dimension, (8) can be written exactly as a linear differential equation as
\[
\frac{d}{dt} \left( (t-kT) \Gamma(\bar{x}'(kT), t-kT) \right) - \bar{A}(t) \left( (t-kT) \Gamma(\bar{x}'(kT), t-kT) \right) = -\left( \bar{A}(t) \bar{x}(kT) + \bar{B}(t) \bar{u}(t) \right) = 0,
\]
whose solution is [13]
\[
(t-kT) \Gamma(\bar{x}'(kT), t-kT) = \int_{t-kT}^{t} e^\left(\int_{t-kT}^{\tau} d\tau' \right) \left( \bar{A}(\tau) \bar{x}(\tau) + \bar{B}(\tau) \bar{u}(\tau) \right) d\lambda.
\]
This leads to the exact discrete-time model [14] as
\[
\Gamma(x_k, T) = \frac{1}{T} \int_{t_k}^{t_{k+1}} e^\left(\int_{t_k}^{\tau} d\tau' \right) \left( \bar{A}(\tau) \bar{x}(\tau) + \bar{B}(\tau) \bar{u}(\tau) \right) d\lambda
\]
where
\[
\Phi(t, \lambda) = 1 + \int_{t}^{\lambda} \bar{A}(\sigma) d\sigma + \int_{t}^{\lambda} \bar{A}(\sigma) \int_{\sigma}^{\lambda} \bar{A}(\tau) d\tau d\sigma + \ldots
\]
\[
+ \int_{t}^{\lambda} \bar{A}(\sigma) \int_{\sigma}^{\lambda} \bar{A}(\tau) \ldots \int_{\tau}^{\lambda} \bar{A}(\xi) d\xi \ldots d\sigma d\sigma_1 + \ldots
\]
Remark 4: When the Taylor series expansion of \( \Gamma(\bar{x}'(kT) + (t-kT) \Gamma(\bar{x}'(kT), t-kT), t) \) is truncated after the first term and noting that \( \Gamma(\bar{x}(t), t) \) is invariant in the interval \([kT, (k+1)T]\), (8) yields
\[
\frac{d}{dt} \left( (t-kT) \Gamma(\bar{x}'(kT), t-kT) \right) = \Gamma(\bar{x}'(kT), kT)
\]
so that
\[
\Gamma(\bar{x}'(kT), t-kT) = \Gamma(\bar{x}'(kT), kT).
\]
This is known as the forward difference model.

Remark 5: When the continuous-time system (1) is an autonomous system \( d \bar{x}(t)/dt = \Gamma(\bar{x}(t)) \), the proposed discrete-time function (10) leads to the discrete-time model for autonomous system proposed in [10]
\[
\delta x_k = \frac{1}{T} \int_{0}^{T} e^{i \omega(t) \delta x_k} d\lambda \Gamma(x_k).
\]
Remark 6: When the continuous-time system (1) is given in the form of a forced nonlinear oscillator with the system function being
\[
\Gamma(\bar{x}(t), t) = \bar{A}(\bar{x}(t)) + \bar{B}(t),
\]
the proposed discrete-time function (10) yields the following discrete-time function:
\[
\Gamma(x_k, T) = \frac{1}{T} \int_{0}^{T} e^{i \omega(t) \delta x_k} d\lambda \Gamma(x_k, \lambda) + \frac{1}{T} \int_{0}^{T} e^{i \omega(t) \delta x_k} \bar{B}(t) \bar{u}(t) d\lambda
\]
where
\[
\bar{A}(t) \bar{x}(kT) + \bar{B}(t) \bar{u}(t) = 0,
\]
which is identical to one proposed in [11].

IV. Example: An Inverted Pendulum Subjected to High-frequency Excitations

Open-loop stabilization of an unstable equilibrium state of an inverted pendulum was shown to be possible in [15], where a high frequency excitation is used in the vertical axis with no feedback. This excited pendulum, shown in Fig. 1, is a nonlinear non-autonomous system, which consists of a point mass \( m \) attached to the top end of a massless rod of length \( l \). The bottom end is periodically excited along the vertical axis with the amplitude and angular frequency of excitation being \( a \) and \( \omega \), respectively. Its equation of motion is given by [15]
\[
d^2 \bar{x}/dt^2 + c \frac{d \bar{x}}{dt} + \left( \frac{g}{l} + \frac{a \omega^2}{l} \cos \omega t \right) \sin \bar{\theta} = 0
\]

where \( c \) is a viscous damping coefficient of the pivot at bottom. The system (37) can be rewritten in a vector form as
\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\rho}
\end{bmatrix} =
\begin{bmatrix}
\rho \\
-m \rho - \left( \frac{g}{l} + \frac{a \omega^2}{l} \cos \omega t \right) \sin \bar{\theta}
\end{bmatrix}
\]

for which the proposed discrete-time model is obtained as
\[
\begin{bmatrix}
\delta \bar{x}_k \\
\delta \bar{\theta}_k
\end{bmatrix} = \frac{1}{T} \int_{0}^{T} \Phi((k+1)T, \lambda) \Gamma(x_k, \lambda) d\lambda,
\]
where
\[
\begin{bmatrix}
\rho_k \\
-m \rho_k - \left( \frac{g}{l} + \frac{a \omega^2}{l} \cos \omega t \right) \sin \bar{\theta}_k
\end{bmatrix}
\]

Figure 1. The inverted pendulum subjected to high-frequency excitation at the base.
Peano-Baker series \( \Phi(z, \lambda) \) is given by (25) with Jacobian matrix being

\[
D \Phi(x_i) = \begin{bmatrix} 0 & 1 \\ -\left( -\frac{g}{l} + \frac{a_c \omega^2}{l} \cos \omega t \right) \cos \theta_i - \frac{c}{ml} \end{bmatrix}. \tag{41}
\]

The forward difference model of continuous-time system (38) is given by

\[
\begin{bmatrix} \delta \theta_i \\ \delta \rho_i \end{bmatrix} = \begin{bmatrix} -\frac{c}{ml} \rho_i - \left( -\frac{g}{l} + \frac{a_c \omega^2}{l} \cos (\omega k T) \right) \sin \theta_i \\ \rho_i \end{bmatrix}. \tag{42}
\]

Simulations have been carried out where \( m, l, c \) are, respectively, \( 2.35 \times 10^{-2} \text{ kg} \), \( 5.37 \times 10^{-3} \text{ m} \) and 0.03. Yabuno et al. [15] have shown that the unstable equilibrium point \( (\theta = 0) \) of the inverted pendulum can be stabilized without feedback by applying a high-frequency excitation with the amplitude of \( a_c = 4.5 \times 10^{-2} \text{ m} \) and the frequency of \( \omega = 35 \text{ Hz} \). Figures 2 to 5 to be given below show the time responses for the first 0.2 seconds, starting from the initial condition of \( \theta_0 = 0.3 \) and \( \dot{\theta}_0 = 0 \). The sum of differences between continuous-time and discrete-time responses is used to assess the error \( ER \), as

\[
ER = T \sum_{k=0}^{i \infty} |\theta(t)|_{t=kT} - \dot{\theta}_i | (43)
\]

where \( t \) is the time length of the simulation run.

From simulations, it was found that, in many cases, only a single term suffices in the calculation of Peano-Baker series. Fig. 2 shows that when the sampling interval is \( T = 0.001 \text{ second} \), the response of the proposed model is very close to that of the continuous-time model even with \( i = 1 \) in Peano-Baker series. The continuous-time response was calculated using the ode45 method (Runge-Kutta, Dormand Prince (4, 5) pair) in Matlab/Simulink. The performance of the forward difference method is not good even at such a small sampling interval; the sampling interval should be reduced to 0.00005s to obtain a response comparable to that of the original continuous-time system. When the sampling interval is increased to 0.0025s (Fig. 3), the proposed model still gives almost exact responses at the sampling instants with \( i = 1 \). When the sampling interval is increased to 0.005s (Fig. 4), the forward difference model becomes unstable, whereas the proposed model still yields stable responses. The response can be improved by increasing \( i \), as shown in Fig. 5 where \( i = 3 \).

Fig. 6 shows the error evaluated by (43) where \( t = 0.2 \text{ s} \). The error becomes smaller as the sampling period does in all cases. The proposed model with \( i = 1 \) has the error whose magnitude is orders of magnitude smaller than that of the forward-difference model, and the difference between the two models becomes larger as the sampling period is reduced. The figure also shows that increasing \( i \) beyond two or three does not reduce the error much further. It takes about 20, 65, 118, and 225 seconds, respectively for \( i = 2, 3, 4, \) and 5, to calculate the coefficients of the Peano-Baker series off-line, as measured by the “cputime” command. The time it takes for Matlab to compute the response in Fig. 4 is 2.18s for the proposed model and 0.95s for the forward difference model.
A method of obtaining an on-line computable discrete-time model has been proposed for a rather general class of nonlinear non-autonomous systems. The proposed method offers an accurate model that is alternative to the popular forward-difference model, which has been practically the only on-line computable technique to such systems. This method is an extension of that introduced previously by the same authors for a special class of non-autonomous nonlinear systems [11]. The proposed discrete-time model can be exact when a certain condition equation can be solved exactly, such as for linear systems. When this condition can be solved approximately an approximate discrete-time model can be derived. The approximate model can always be obtained as long as the Jacobian matrix exists for the continuous-time system and Peano-Baker series can be computed. As an example, the proposed method was applied to open-loop control of an inverted pendulum under high-frequency excitation. Simulations show that the proposed model gives good performances even for relatively large sampling intervals. It was also found that, for sufficient small sampling intervals, only a single term is needed in the Peano-Baker series computation. When a larger sampling interval is used, higher order Peano-Baker series can be used to improve the performance of the proposed method.

While the present paper does not address a particular controller design methodology, it proposes an accurate discretization technique that is applicable to a rather general class of control algorithms, which may be nonlinear and time-varying for online computations, such as discretization of a continuous-time adaptive control law.

V. CONCLUSIONS

A method of obtaining an on-line computable discrete-time model has been proposed for a rather general class of nonlinear non-autonomous systems. The proposed method offers an accurate model that is alternative to the popular forward-difference model, which has been practically the only on-line computable technique to such systems. This method is an extension of that introduced previously by the same authors for a special class of non-autonomous nonlinear systems [11]. The proposed discrete-time model can be exact when a certain condition equation can be solved exactly, such as for linear systems. When this condition can be solved approximately an approximate discrete-time model can be derived. The approximate model can always be obtained as long as the Jacobian matrix exists for the continuous-time system and Peano-Baker series can be computed. As an example, the proposed method was applied to open-loop control of an inverted pendulum under high-frequency excitation. Simulations show that the proposed model gives good performances even for relatively large sampling intervals. It was also found that, for sufficient small sampling intervals, only a single term is needed in the Peano-Baker series computation. When a larger sampling interval is used, higher order Peano-Baker series can be used to improve the performance of the proposed method.

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